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The Shape of the Earth

Navigation involves moving along or above the surface of the Earth (or other planet), and in order to understand navigation we need to understand the Earth's shape. Early pilots followed roads or railways and could ignore this, and many pilots fly this way today. But modern aviation is a global enterprise, and with the introduction of the Global Positioning System (GPS) modern pilots need to understand how charts are constructed, how coordinates of ground positions are determined, and more about the nature of the Earth's surface.¹

Why do we need to know this? As you probably know, we describe a position on the Earth's surface using latitude and longitude, and we also use these coordinates to describe the places we want to go. Since we want to know how far and which way to go, we (or our computers) need to understand the geometry of the planet. And to know where we are, we typically measure the distance and direction we have gone, so we need to know how to interpret these measurements to get latitude and longitude. Thus, a detailed model of the Earth's surface is a basic need for the practice of navigation.

In this chapter we will study the Earth's shape, how the shape is known, how the size of the Earth is known, what this means about charts, the meaning of latitude and longitude, and the importance of knowing the actual (rather than theoretical) shape of the Earth. In

Now imagine moving on a *plane*: There are evidently an infinite number of directions of possible movement, but we can single out two of them and determine every direction in terms of those two. We still have to pick an origin O , and a unit distance. But now we can go to the right from O (you can imagine a horizontal line drawn through O), and we can also go up from O (imagine a vertical line). In this coordinate system, left and down are negative (see Figure 1-2).

To reach any point on the line, we specify the number of (positive or negative) units to move from O , and on the plane we specify the number of left-right units and the number of up-down units. Thus the line has one dimension, and the plane has two. We imagine in both cases that we can go an unlimited distance in any direction; this may not be practical, but it has the philosophical advantage of pushing some of the difficulties out of sight.

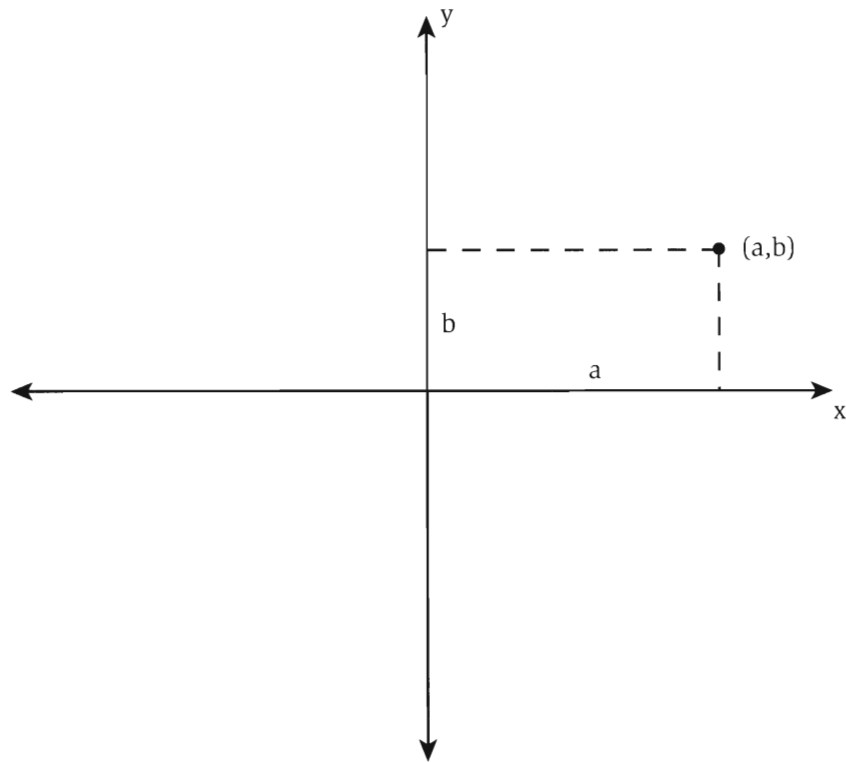


Figure 1-2. Coordinates on a plane.

order to do this we will review some simple mathematical tools that will also be helpful in later work.

One theme of this book is that navigation is a combination of action and contemplation. You need to do some calculations (the contemplation part), but then you need to go out, get moving, and see where you end up (the action part). A prudent navigator strives to perfect both.

The Earth Is Round

Virtually everyone believes that the Earth is “round.” In the twenty-first century, we have been lucky enough to see photographs taken from space, and the regular use of artificial satellites for communications, geodesy, surveillance, remote sensing, and navigation confirms that the round Earth model works, at least to a certain level of accuracy.

In the United States, this idea is popularly attributed to Columbus, but the idea of a spherical Earth is much older. Aristotle reasoned that the Earth was round based on common observation: a departing ship’s hull disappears over the horizon before its masts, and this happens in every direction (Wilford). The stars change as one moves north and south. The shadow of the Earth cast during a lunar eclipse was further evidence.²

The Greeks liked to measure, and given the model of the Earth as a sphere, it was natural to determine its size. A sphere has one size parameter, its *radius*. Another size parameter is the *circumference*. If you know one of these, then you know the other, since the circumference of a circle is 2π times the radius, where π is about 3.14159... or $22/7$. So to measure the Earth, you need to measure its radius. How is it possible to measure such a vast distance in space?

Knowing the circumference of the Earth means that we know how much range to build into our airplanes. It means that we can make inertial navigation (see Chapter 5) work reliably, and it means that we can interpret the signals from GPS satellites accurately.

Eratosthenes, a Greek polymath, gave the first known estimate of the circumference (and hence the radius) of the Earth. In general,